

Analysis of Portuguese Life Tables

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SUMMARY

In 1995 life tables commissioned by the Portuguese Insurance Institute were obtained. These life tables were then validated and are now updated. The updated tables are used to compare mortality for men and women.

Key words: Life tables; Population mortality; Portuguese case.

1. Introduction

Portuguese life tables are based on national censuses and have been published every 10 years since the beginning of the 20th century, issued for males and females separately and for the whole population; see Cruz Alves (1997).

In 1995 the Faculty of Science and Technology of Nova University of Lisbon published the first “auto-corrective” mortality tables, by sex, known in Portugal (Mexia and Corte-Real (1995)) at the request of the Portuguese Insurance Institute. At the time, the adjustment of these tables was validated by Bernardino (1995) and Cruz Alves (1997) who showed that the new tables were better adjusted than others currently used in Portugal.

As seen in Chiang (1984), the standard techniques for life table construction rest on the probabilities q_x of a man or woman of age x dying before attaining age $x+1$.

A preliminary study of the evolution of the probabilities q_x presented in Mexia and Corte-Real (1995) showed that they had stabilized or at least smoothly

evolved. An update of these probabilities was carried out, using a procedure described later on, in order to update the Portuguese life tables. We present the new tables and discuss some demographic implications.

2. Life tables

A life table is a concise way of showing the probabilities of a member of a particular population living to or dying at a particular age. It summarizes the mortality experience of a population as determined by historical data and prior opinion concerning that population.

Life tables play a central role in the Actuarial Sciences. They are primarily used for the calculation of life insurance premiums.

Life tables are usually constructed separately for men and women because of their substantially different mortality rates. Other characteristics can also be used to distinguish different risks, such as smoking-status, occupation and socio-economic class.

2.1. The survival function and some sample calculations

Let us consider a newborn child. This newborn's age-at-death, X , is a continuous type random variable. Let $F_X(x)$ denote the distribution function of X ,

$$F_X(x) = P(X \leq x), \quad x \geq 0,$$

and set

$$s(x) = 1 - F_X(x) = P(X > x), \quad x \geq 0.$$

We assume that $F_X(0) = 0$, which implies $s(0) = 1$. The function $s(x)$ is called the survival function. Then for a positive x , $s(x)$ is the probability of a newborn attaining age x . The distribution of X can be defined by specifying either the function $F_X(x)$ or the survival function $s(x)$, see Bowers *et al.* (1997).

Using these functions we get:

- $P(x < X \leq y) = F_X(y) - F_X(x) = s(x) - s(y)$ this is the probability of a newborn dying between ages x and y ($x < y$).

- $P(x < X \leq y | X > x) = \frac{F_x(y) - F_x(x)}{1 - F_x(x)} = \frac{s(x) - s(y)}{s(x)}$ this is the conditional probability of a newborn dying between ages x and y , given that it survived to age x .

Before updating the Portuguese life tables we introduce some useful functions. Firstly $T(x) = X - x$ will be the life-span of a person that attained age x and:

- ${}_t q_x = P[T(x) \leq t]$, $t \geq 0$ is the probability that a person with age x will die within t years;
- ${}_t p_x = 1 - {}_t q_x = P[T(x) > t]$, $t \geq 0$ is the probability that a person with age x will attain age $x + t$;

Namely we will have $T(0) = X$, ${}_x p_0 = s(x)$, $x \geq 0$ and:

- $q_x = {}_1 q_x$ will be the probability of an individual with age x dying before his $(x-1)$ th birthday;
- $p_x = {}_1 p_x = 1 - q_x$ will be the probability of surviving from age x to age $x+1$;

Beside the probabilities q_x , in the life tables we have two more columns, assigned to:

- l_x - the number of people who survive to age x starting usually with $l_0 = 100.000$. Since $l_{x+1} = l_x(1 - q_x) = l_x p_x$ we have ${}_t p_x = \frac{l_{x+t}}{l_x}$.
- $e_x = \sum_{t=1}^{\infty} {}_t p_x$ - the life expectancy at age x . This value is calculated by adding up the probabilities of surviving to every age, i.e. the expected number of complete years lived (one may think of it as the number of birthdays they will celebrate).

These special functions are taken for both sexes.

3. Updating the tables

3.1. Method Used

We now describe how the life tables were updated.

Using the data on:

- death of Portuguese men and women from 1917 to 1992;
- birth of Portuguese men and women from 1960 to 1992,

the probabilities q_x for both sexes were obtained for every year from 1940 to 1992.

For instance, for women and ages 11 and 42 we obtained the graphs which are shown in Figure 1.

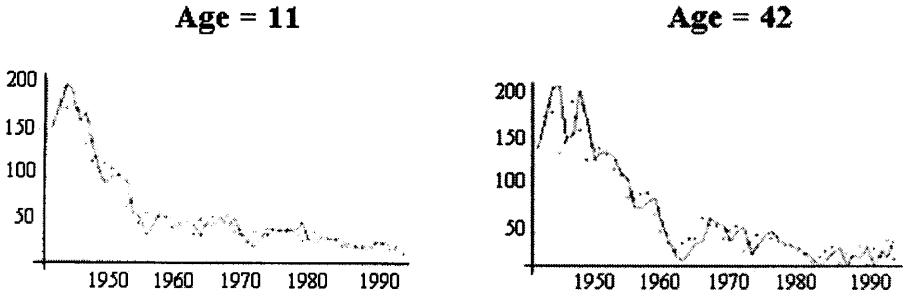


Figure 1. Mortality in males being 11 (on the left) and 42 (on the right) years old. Dots represent the chrono-series and the lines the forecast

We see in these two graphs a stabilization of the probabilities q_x from 1970 onwards. This stabilization may be observed for other ages and also for men.

Thus we decided to use the Holt double exponential method to update to the present the values of the probabilities q_x . This method minimizes the sum of squares of the errors of forecasting. An extrapolation was then carried out to predict future mortality rates, where:

- the origin of time is placed at the present;
- the weights of the observations decrease geometrically as we go back in time.

Hence this method stresses the importance of recent observations and gives less importance to observations that occurred longer ago.

The problem of forecasting in statistics has been studied deeply, see for example Chatfeld (1989) or Murteira *et al.* (1993).

3.2. Results and updated results

In our updating we also used polynomial splines of degrees 4 and 6 to smooth the peaks in the original time series. To reduce the distances between forecasted and adjusted values we opted, after some attempts, to divide the series in two sub-series; one up to 65 years and another from 66 to 105 years old.

In Table 1 we present, for men, the probabilities q_x used by Mexia and Corte-Real (1995) and those obtained through the updating. The corresponding probabilities for women are presented similarly in Table 2.

The evolution of these probabilities for men and women are presented in Figures 2 and 3 respectively.

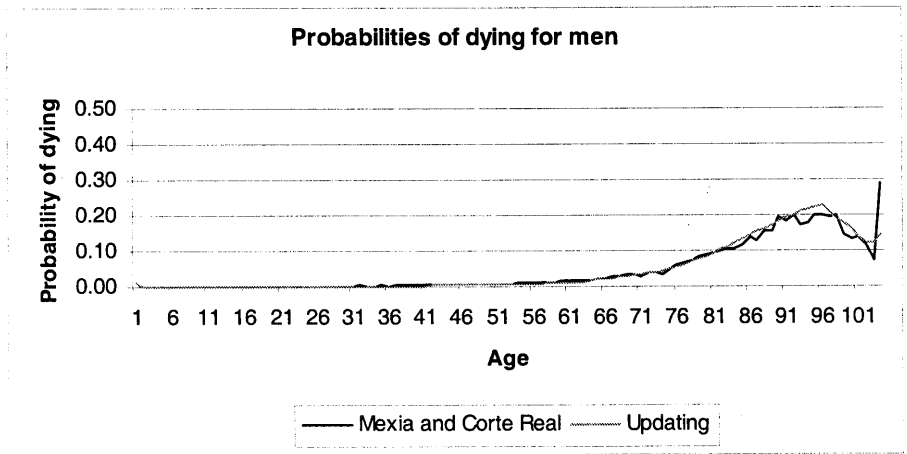


Figure 2. Probabilities of dying for men as used by Mexia and Corte Real and those obtained through the updating, based on Table 1

Table 1. Probabilities of dying ($\times 10^{-3}$) as used by Mexia and Corte Real (1995) and those obtained through the updating, for men.

Age	Mexia and Corte Real	Updating	Age	Mexia and Corte Real	Updating	Age	Mexia and Corte Real	Updating
0	9.058	9.058	36	3.122	2.554	72	38.818	41.362
1	0.987	0.789	37	3.172	2.597	73	34.967	45.911
2	0.823	0.647	38	3.439	2.646	74	50.687	51.049
3	0.493	0.542	39	3.629	2.703	75	62.529	56.787
4	0.833	0.472	40	3.522	2.770	76	66.969	63.128
5	0.334	0.433	41	3.429	2.850	77	73.117	70.062
6	0.342	0.421	42	3.538	2.944	78	81.388	77.574
7	0.456	0.433	43	4.216	3.057	79	86.733	85.635
8	0.412	0.466	44	3.335	3.191	80	96.770	94.209
9	0.125	0.517	45	4.471	3.349	81	99.097	103.250

10	0.331	0.583	46	4.744	3.534	82	104.591	112.703
11	0.274	0.662	47	5.205	3.750	83	107.007	122.501
12	0.395	0.751	48	5.321	4.000	84	117.971	132.570
13	0.576	0.848	49	4.990	4.288	85	138.282	142.825
14	0.577	0.950	50	5.838	4.619	86	129.959	153.173
15	0.913	1.057	51	6.325	4.996	87	156.199	163.508
16	1.380	1.165	52	7.679	5.423	88	156.315	173.719
17	1.816	1.275	53	8.479	5.905	89	192.162	183.681
18	1.893	1.383	54	9.344	6.447	90	180.571	193.262
19	1.876	1.489	55	10.525	7.053	91	197.877	202.320
20	1.750	1.592	56	11.133	7.728	92	170.350	210.704
21	1.937	1.691	57	11.733	8.478	93	180.434	218.251
22	1.742	1.784	58	12.504	9.308	94	197.320	224.791
23	1.953	1.872	59	14.079	10.222	95	200.171	230.143
24	2.217	1.955	60	16.221	11.228	96	191.908	205.473
25	2.441	2.031	61	16.239	12.330	97	197.952	190.485
26	2.252	2.101	62	16.894	13.535	98	144.928	177.026
27	2.248	2.164	63	18.742	14.848	99	136.054	160.556
28	1.991	2.222	64	21.472	21.628	100	137.256	141.335
29	2.329	2.274	65	21.616	23.029	101	114.943	124.424
30	2.554	2.321	66	27.371	24.583	102	72.993	119.684
31	2.884	2.364	67	26.850	26.376	103	289.157	141.775
32	2.719	2.404	68	32.079	28.489	104	408.163	210.157
33	2.672	2.441	69	32.290	30.989	105	476.190	349.090
34	2.985	2.478	70	25.009	33.936	106	1000.000	1000.000
35	2.772	2.515	71	38.835	37.380			

Table 2. Probabilities of dying ($\times 10^{-3}$) as used by Mexia and Corte Real (1995) and those obtained through the updating, for women

Age	Mexia and Corte Real	Updating	Age	Mexia and Corte Real	Updating	Age	Mexia and Corte Real	Updating
0	6.887	6.887	36	0.993	0.715	72	18.273	19.181
1	0.458	0.713	37	1.240	0.743	73	19.624	20.835
2	0.359	0.572	38	0.707	0.776	74	29.109	32.258
3	0.552	0.455	39	1.406	0.816	75	36.733	32.539
4	0.353	0.359	40	1.339	0.863	76	40.500	38.234
5	0.426	0.282	41	1.613	0.920	77	44.644	46.031
6	0.437	0.223	42	1.512	0.986	78	52.702	53.898
7	0.166	0.180	43	1.857	1.065	79	56.647	60.797
8	0.174	0.150	44	2.326	1.158	80	68.139	66.434
9	0.314	0.132	45	2.138	1.266	81	75.724	71.029
10	0.288	0.125	46	2.663	1.392	82	80.115	75.122
11	0.106	0.127	47	2.058	1.537	83	82.167	79.405
12	0.088	0.137	48	2.011	1.703	84	101.625	84.581
13	0.225	0.152	49	2.316	1.893	85	29.165	91.254
14	0.358	0.173	50	2.849	2.109	86	117.998	99.849
15	0.241	0.198	51	3.073	2.353	87	132.804	110.558
16	0.238	0.226	52	4.254	2.628	88	147.623	123.318
17	0.227	0.256	53	3.882	2.936	89	161.140	137.809
18	0.397	0.288	54	3.813	3.280	90	163.228	153.498
19	0.460	0.319	55	4.366	3.662	91	166.433	169.694

20	0.461	0.351	56	4.800	4.086	92	178.531	185.640
21	0.336	0.383	57	5.157	4.554	93	171.503	200.635
22	0.718	0.413	58	5.178	5.070	94	201.782	214.183
23	0.509	0.442	59	6.319	5.636	95	207.558	226.168
24	0.579	0.469	60	8.539	6.255	96	215.219	237.064
25	0.598	0.494	61	6.708	6.932	97	205.770	248.166
26	0.605	0.518	62	7.372	7.669	98	210.277	261.858
27	0.636	0.540	63	8.446	8.470	99	265.922	281.902
28	0.664	0.560	64	9.630	9.339	100	161.832	313.763
29	0.629	0.579	65	11.527	10.278	101	152.577	364.959
30	0.672	0.597	66	11.934	11.293	102	127.660	445.437
31	0.673	0.615	67	11.664	12.387	103	284.501	567.985
32	0.705	0.632	68	14.736	13.563	104	403.042	748.665
33	0.746	0.650	69	15.686	14.827	105	456.140	879.310
34	1.125	0.669	70	18.303	16.181	106	1000.000	1000.000
35	0.982	0.690	71	21.403	17.631			

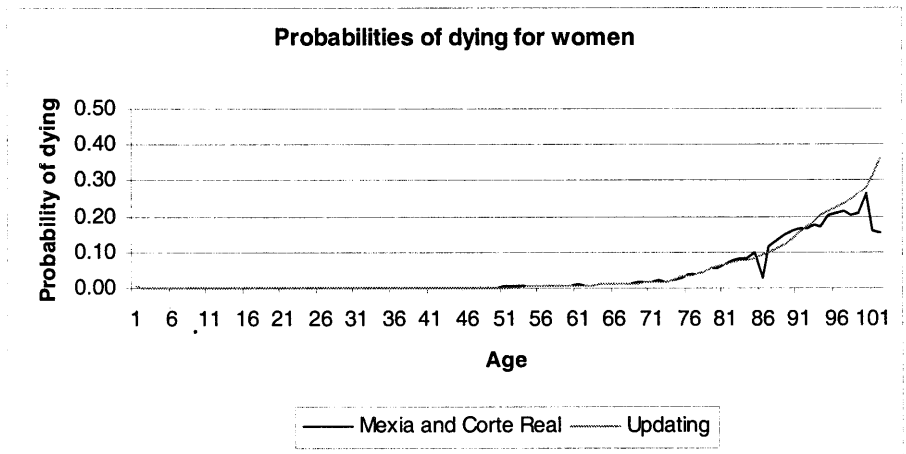


Figure 3. Updated probabilities of dying as used by Mexia and Corte Real (1995) and those obtained through the updating, for women, based on Table 2

We observe that our technique led to a smoothing of the evolution of q_x series as is generally recommended; see Chiang (1984).

3.3. Life expectancy

Life expectancy for a sex and age is the corresponding expected number of years of life. This expectancy changes with time. The resulting tables for the Portuguese population have been presented. Although perfectly adjusted life tables tend to be outdated in themselves, the idea of constructing auto-correlative life tables has been introduced. The basis for this construction is to use forecast

techniques, and also auto-correlatives, for natalities and mortalities for the two sexes and different ages.

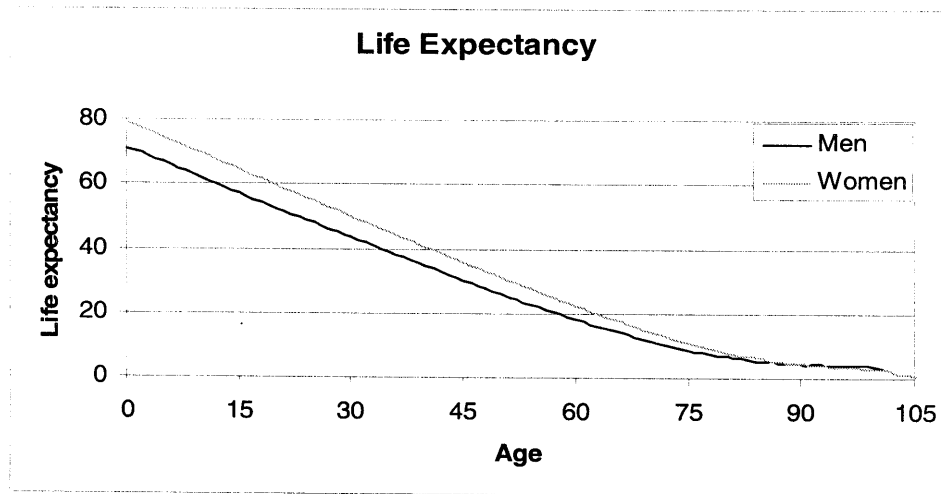


Figure 4. Life expectancy by age and sex

Figure 4 shows the life expectancy for men and women. We can see that the life expectancy for women is almost always superior to the life expectancy for men. Only men aged more than 95 years old had a higher life expectancy than the corresponding women.

Although there are few men at this age because they die before 95 more often than women, they are possibly stronger and more resistant. In the appendix we present the life tables with the probability of dying, the proportion of original individuals still alive and the remaining life expectancy by age and sex.

3.4. Some comparative analysis

We present now some comparative analysis between males and females to try to find any significant differences.

Probability of dying between 5 and 50

Figure 5 presents the comparison between probabilities of dying for men and women with ages between 5 and 50 years. The differences between men and women are vast. Between 15 and 20 years the probability of dying for males grows significantly, probably because some risk attitudes are more common in

men than in women. Between 40 and 45 years we can see a significant growth in the probabilities of dying, as much in men as in women.

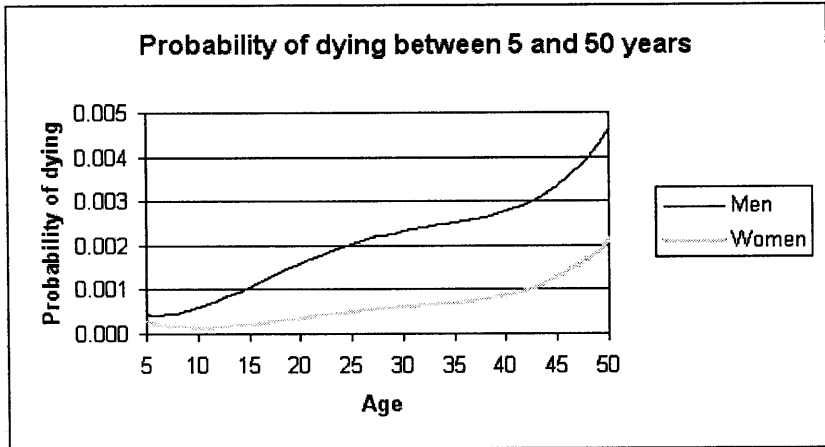


Figure 5. Probabilities of dying between 5 and 50 years

Probability of dying between 65 and 105

In Figure 6 we present the comparison between probabilities of dying for men and women with ages between 65 and 105.

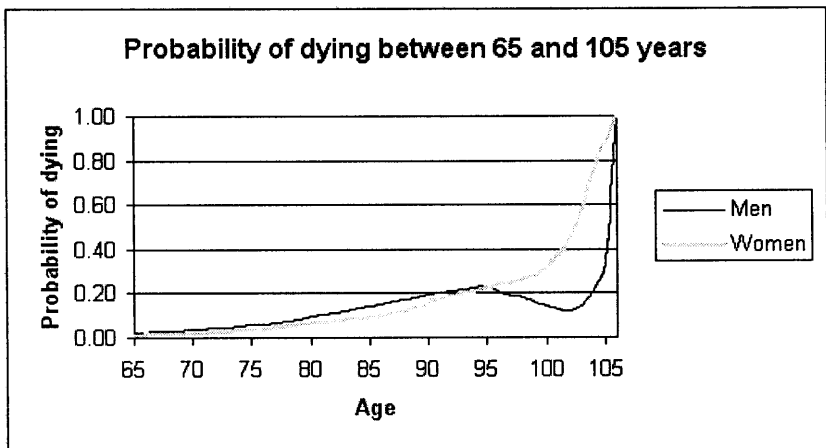


Figure 6. Probabilities of dying between 65 and 105 years

We can see that the probability of dying, up to 95 years, is greater for men than for women but after 95 years (as we concluded from the analysis of life expectancy) men were less likely to die (the number of men is smaller than the number of women as we can see in the tables in the appendix).

4. Final comments

We conclude by pointing out that:

- The probability of dying is greater for men aged less than 95 years. The opposite is true of life expectancy;
- Between 15 and 25 years, the probability of dying for men is about 4.5 times greater than the probability of dying for women;
- The probability of dying between 41 and 50 years is 1.4 times larger than the probability of dying between 31 and 40 for men and 2.0 times larger for women;
- The life expectancy of women at birth is almost 8 years longer than that for men.

Appendix

Tabela 3. Life table for Portuguese males

Interval age (in years) from x to x+1	Probability of dying in the interval (x,x+1) 1000 qx	Number of people alive with age x lx	Life expectancy at age x ex	Interval age (in years) from x to x+1	Probability of dying in the interval (x,x+1) 1000 qx	Number of people alive with age x lx	Life expectancy at age x ex
0	9.058	100000	70.96	54	6.447	86406	22.67
1	0.789	99094	70.61	55	7.053	85599	21.88
2	0.647	98996	69.68	56	7.728	84698	21.12
3	0.542	98915	68.74	57	8.478	83755	20.35
4	0.472	98866	67.77	58	9.308	82772	19.59
5	0.433	98784	66.83	59	10.222	81737	18.84
6	0.421	98751	65.85	60	11.228	80587	18.11
7	0.433	98717	64.85	61	12.330	79279	17.41
8	0.466	98672	63.88	62	13.535	77992	16.70
9	0.517	98631	62.91	63	14.848	76674	15.98
10	0.583	98619	61.92	64	21.628	75237	15.29
11	0.662	98586	60.94	65	23.029	73622	14.63
12	0.751	98559	59.95	66	24.583	72030	13.95
13	0.848	98520	58.98	67	26.376	70059	13.34
14	0.950	98464	58.01	68	28.489	68178	12.71
15	1.057	98407	57.04	69	30.989	65991	12.13
16	1.165	98317	56.10	70	33.936	63860	11.54

17	1.275	98181	55.17	71	37.380	62263	10.83
18	1.383	98003	54.27	72	41.362	59845	10.27
19	1.489	97818	53.38	73	45.911	57522	9.68
20	1.592	97634	52.48	74	51.049	55510	9.03
21	1.691	97463	51.57	75	56.787	52697	8.52
22	1.784	97274	50.67	76	63.128	49402	8.09
23	1.872	97105	49.76	77	70.062	46093	7.67
24	1.955	96915	48.86	78	77.574	42723	7.27
25	2.031	96700	47.96	79	85.635	39246	6.91
26	2.101	96464	47.08	80	94.209	35842	6.57
27	2.164	96247	46.19	81	103.250	32374	6.28
28	2.222	96031	45.29	82	112.703	29165	5.97
29	2.274	95840	44.38	83	122.501	26115	5.66
30	2.321	95616	43.49	84	132.570	23321	5.34
31	2.364	95372	42.60	85	142.825	20569	5.05
32	2.404	95097	41.72	86	153.173	17725	4.87
33	2.441	94839	40.83	87	163.508	15421	4.59
34	2.478	94585	39.94	88	173.719	13013	4.44
35	2.515	94303	39.06	89	183.681	10979	4.27
36	2.554	94041	38.17	90	193.262	8869	4.28
37	2.597	93748	37.29	91	202.320	7267	4.22
38	2.646	93450	36.41	92	210.704	5829	4.27
39	2.703	93129	35.54	93	218.251	4836	4.14
40	2.770	92791	34.66	94	224.791	3964	4.05
41	2.850	92464	33.79	95	230.143	3182	4.05
42	2.944	92147	32.90	96	205.473	2545	4.06
43	3.057	91821	32.02	97	190.485	2056	4.03
44	3.191	91434	31.16	98	177.026	1649	4.03
45	3.349	91129	30.26	99	160.556	1410	3.71
46	3.534	90722	29.40	100	141.335	1218	3.29
47	3.750	90291	28.54	101	124.424	1051	2.81
48	4.000	89821	27.69	102	119.684	930	2.18
49	4.288	89343	26.83	103	141.775	862	1.35
50	4.619	88898	25.97	104	210.157	613	0.90
51	4.996	88379	25.12	105	349.090	363	0.52
52	5.423	87820	24.28	106	1000.000	190	0.48
53	5.905	87145	23.47				

Tabela 4. Life table for Portuguese females

Interval age (in years) from x to x+1	Probability of dying in the interval (x,x+1) 1000 qx	Number of people alive with age x lx	Life expectancy at age x ex	Interval age (in years) from x to x+1	Probability of dying in the interval (x,x+1) 1000 qx	Number of people alive with age x lx	Life expectancy at age x ex
0	6.887	100000	78.73	54	3.280	94016	27.56
1	0.713	99311	78.28	55	3.662	93657	26.66
2	0.572	99266	77.32	56	4.086	93249	25.78
3	0.455	99230	76.34	57	4.554	92801	24.90
4	0.359	99175	75.39	58	5.070	92322	24.03
5	0.282	99140	74.41	59	5.636	91844	23.16
6	0.223	99098	73.44	60	6.255	91264	22.30
7	0.180	99055	72.48	61	6.932	90485	21.50

8	0.150	99038	71.49	62	7.669	89878	20.64
9	0.132	99021	70.50	63	8.470	89215	19.80
10	0.125	98990	69.52	64	9.339	88462	18.96
11	0.127	98962	68.54	65	10.278	87610	18.15
12	0.137	98951	67.55	66	11.293	86600	17.36
13	0.152	98942	66.56	67	12.387	85566	16.57
14	0.173	98920	65.57	68	13.563	84568	15.77
15	0.198	98885	64.59	69	14.827	83322	15.00
16	0.226	98861	63.61	70	16.181	82015	14.24
17	0.256	98837	62.63	71	17.631	80514	13.51
18	0.288	98815	61.64	72	19.181	78791	12.80
19	0.319	98776	60.66	73	20.835	77351	12.04
20	0.351	98730	59.69	74	32.258	75833	11.28
21	0.383	98685	58.72	75	32.539	73626	10.62
22	0.413	98652	57.74	76	38.234	70921	10.02
23	0.442	98581	56.78	77	46.031	68049	9.45
24	0.469	98531	55.81	78	53.898	65011	8.89
25	0.494	98474	54.84	79	60.797	61585	8.38
26	0.518	98415	53.87	80	66.434	58096	7.89
27	0.540	98355	52.91	81	71.029	54137	7.46
28	0.560	98293	51.94	82	75.122	50038	7.07
29	0.579	98227	50.98	83	79.405	46029	6.69
30	0.597	98165	50.01	84	84.581	42247	6.29
31	0.615	98100	49.04	85	91.254	37954	6.00
32	0.632	98033	48.07	86	99.849	36847	5.18
33	0.650	97964	47.11	87	110.558	32499	4.87
34	0.669	97891	46.14	88	123.318	28183	4.62
35	0.690	97781	45.20	89	137.809	24023	4.42
36	0.715	97685	44.24	90	153.498	20152	4.27
37	0.743	97588	43.28	91	169.694	16862	4.10
38	0.776	97467	42.34	92	185.640	14056	3.92
39	0.816	97398	41.37	93	200.635	11546	3.77
40	0.863	97261	40.43	94	214.183	9566	3.55
41	0.920	97131	39.48	95	226.168	7636	3.45
42	0.986	96974	38.54	96	237.064	6051	3.35
43	1.065	96828	37.60	97	248.166	4749	3.27
44	1.158	96648	36.67	98	261.858	3772	3.12
45	1.266	96423	35.76	99	281.902	2978	2.95
46	1.392	96217	34.83	100	313.763	2186	3.02
47	1.537	95961	33.93	101	364.959	1833	2.60
48	1.703	95763	33.00	102	445.437	1553	2.07
49	1.893	95571	32.06	103	567.985	1355	1.37
50	2.109	95349	31.14	104	748.665	969	0.92
51	2.353	95078	30.23	105	879.310	579	0.54
52	2.628	94786	29.32	106	1000.000	315	0.50
53	2.936	94382	28.45				

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